Chapter 4 Practice Test

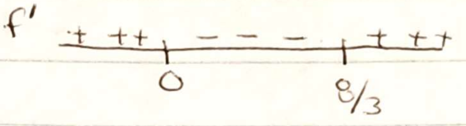
1. $f'(x)=3x^2-8x$

$0=3x^2-8x$

$0=x(3x-8)$

$x=0$

$x=\frac{8}{3}$



$$\boxed{\begin{array}{ll} f(0)=0 & \text{maximum} \\ f(\frac{8}{3})=-\frac{256}{27} & \text{minimum} \end{array}}$$

2. $f''=3x^2-4$

$0=3x^2-4$

$4=3x^2$

$\pm\frac{2}{\sqrt{3}}=x$

$$\boxed{\begin{array}{l} \text{Inflection Points } x=\frac{-2\sqrt{3}}{3} \text{ and } \frac{2\sqrt{3}}{3} \\ \text{Concave Up: } (-\infty,\frac{-2\sqrt{3}}{3})\ (\frac{2\sqrt{3}}{3},\infty) \\ \text{Concave Down: } (\frac{-2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}) \end{array}}$$

3. Find slope between the endpoints.

$f(0)=0$

$f(4)=8$ slope $=\frac{8-0}{4-0}=2$

Set derivative equal to slope

$f'(x)=2x-2$

$2=2x-2$

$4=2x$

$2=x$

$$\boxed{\text{The mean value is located at } x=2.}$$

4. $f(x) = \dfrac{2x^2+1}{(x-5)(x+3)}$

$\boxed{\text{Vertical Asymptotes at } x=5 \text{ and } x=-3}$

Take limit at infinity to find horizontal asymptote.

$\lim_{x\to\infty} \dfrac{2\infty^2}{\infty^2} = 2$ $\boxed{y=2 \text{ Horizontal asymptote}}$

5. $V = x^3$

$\frac{dV}{dx} = 3x^2$

$dV = 3x^2\,dx$

$dV = 3(8 \text{ inches})(.25 \text{ inches})$

$\boxed{dV = 48 \text{ in}^3}$

6. $y(1.5) = 15.1875$

$y(1.6) = 19.6608$

$\boxed{\Delta y = 4.4733}$

$\frac{dy}{dx} = 12x^3$

$dy = 12x^3 \cdot dx$

$dy = 12(1.5)^3(.1)$

$\boxed{dy = 4.05}$

7. Surface Area: $420 = 2x^2 + 4xy$

$420-2x^2=4xy$

$\frac{420-2x^2}{4x}=y$

Volume: $V = x^2y$

$V = x^2\left(\frac{420-2x^2}{4x}\right) = 105x - .5x^3$

$\frac{dV}{dx} = 105 - 1.5x^2$

$0 = 105 - 1.5x^2$

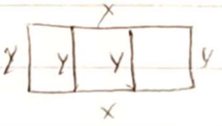
$105 = 1.5x^2$

$70 = x^2$

$\boxed{8.367 = x}$

$y = \frac{420-2(8.367)^2}{4(8.367} = \boxed{8.367}$

b. $\boxed{\text{Volume} = x^2y = 585.66 \text{ in}^3}$



8. $900 = 2x + 4y$

$900 - 2x = 4y$

$\frac{900-2x}{4} = y$

$A = x \cdot y = x\left(\frac{900-2x}{4}\right) = 225x - \frac{1}{2}x^2$

$\frac{dA}{dx} = 225 - x$

$0 = 225 - x$

$\boxed{225' = X}$

$y = \frac{900-2(225)}{4} = \boxed{112.5 \text{ feet}}$

8. a. $0 = \dfrac{x^2}{x^2-4}$

$0 = x$ $\boxed{x\text{-intercept } (0,0)}$

b. $y = \dfrac{0^2}{0^2-4} = 0$

$\boxed{y\text{-intercept } (0,0)}$

c. Vertical asymtotes $0 = x^2 - 4$

$\pm 2 = x$

$\boxed{\begin{array}{ll}\text{V.A at} & x = -2 \\ & x = 2\end{array}}$

d. Horizontal asymptotes

$\lim\limits_{x \to \infty} \dfrac{\infty^2}{\infty^2 - 4} = 1$

$\boxed{\text{H.A at } y = 1}$

e. Slant asymptote : none

f. Domain: $\boxed{x \neq -2 \text{ or } 2}$

g. Range: ~~$y \neq 1$~~ $\boxed{y: (-\infty, 0] \cup (1, \infty)}$

graph it to find range.

h. $$y' = \frac{(x^2-4)(2x) - (x^2)(2x)}{(x^2-4)^2}$$

$$y' = \frac{2x^3 - 8x - 2x^3}{(x^2-4)^2} = \boxed{\frac{-8x}{(x^2-4)^2}}$$

i. $$0 = \frac{-8x}{(x^2-4)^2}$$

C. N. $$\boxed{\begin{array}{c} x = 0 \\ x = -2 \\ x = 2 \end{array}}$$

j. f' ++++ | ++++ | − − − | − − −

-2 0 2

Increasing: $(-\infty, -2)$ $(-2, 0)$
Decreasing: $(0, 2)$ $(2, \infty)$

k. $$y'' = \frac{(x^2-4)^{\cancel{2}}(-8) - (-8x)(2)\cancel{(x^2-4)}(2x)}{(x^2-4)^{\cancel{4}3}}$$

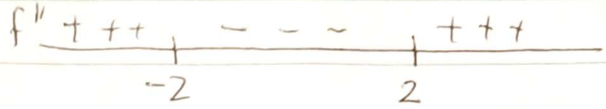
$$y'' = \frac{-8x^2 + 32 + 32x^2}{(x^2-4)^3} = \boxed{\frac{24x^2 + 32}{(x^2-4)^3}}$$

L. $$0 = \frac{24x^2 + 32}{(x^2-4)^3}$$

$0 = 24x^2 + 32$
$-32 = 24x^2$
$\emptyset$

Inflection points $x = -2$
$x = 2$

m.



Concave up: $(-\infty, -2)$ $(2, \infty)$

Concave down: $(-2, 2)$