







Polar Equations of Conics and Other Shapes

Lines:

-  Horizontal line $r = \frac{k}{\sin \theta}$
-  Vertical line $r = \frac{k}{\cos \theta}$
-  Diagonal line $\theta = \text{an angle}$ (If r is not specified, it is equal to all real numbers.)


Circles:


-  Center at pole $r = a$
-  Center on x-axis $r = a \cos \theta$
-  Center on y-axis $r = a \sin \theta$


Ellipse, Parabola, or Hyperbola:

Horizontal $r = \frac{ep}{1 \pm e \cos \theta}$

Vertical $r = \frac{ep}{1 \pm e \sin \theta}$

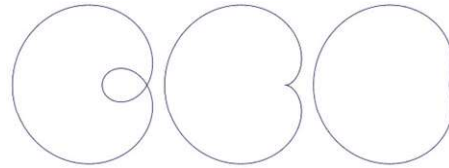
 If $e < 1$, it is an ellipse.

 If $e = 1$, it is a parabola.

 If $e > 1$, it is a hyperbola.

Limaçons:

$r = a \pm b \cos \theta$ or $r = a \pm b \sin \theta$



If $|a| < |b|$, it has two loops.

If $|a| = |b|$, it has one loop and looks like a heart. It's called a cardioid.

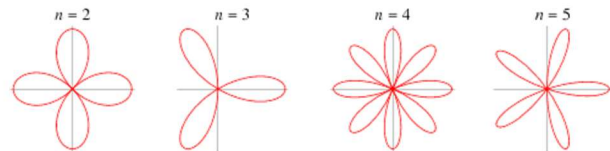
If $|a| > |b|$, it has one flattened loop.

Rose Curves:

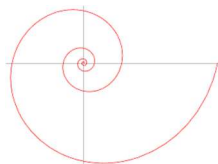
$r = a \cos n\theta$ or $r = a \sin n\theta$ ($n \geq 2$)

If n is odd, the curve has n petals.

If n is even, the curve has $2n$ petals.



Spiral: $r = \frac{a\theta}{\text{Angle}} \pm b$



Section Exercises 12.3

In Exercises 1 – 8, use your calculator to graph each function.

- Identify the function as a circle, ellipse, parabola, hyperbola, limaçon, or rose curve.
- For rose curves, indicate the number of petals. For limaçons, indicate whether it has one loop, has two loops, or it's a cardioid.

1. $r = \frac{6}{1 - \cos \theta}$

2. $r = 3$

3. $r = \frac{7}{1 - 2 \sin \theta}$

4. $r = 4 - 2 \cos \theta$

5. $r = 3 \sin \theta$

6. $r = 2 \cos 3\theta$

7. $r = 2 - 3 \sin \theta$

8. $r = \frac{9}{2 - \sin \theta}$

In Exercises 9 – 15, write the equation for the function that is described.

- A circle centered at the pole with a radius of 5.
- A circle centered on the y-axis with a radius of 2.
- A parabola symmetric to the x-axis with $p = 2$ that opens to the left.
- A limaçon with two loops. $b = 4$
- A cardioid with $b = 2$.
- A rose curve with eight petals. Each petal is 3 units long.
- A rose curve with five petals. Each petal is 2 units long.