Geometry Facts to Know

Types of Angles and Angle Pairs

- Types of Angles:

  Acute – less than 90°

  Right – Equal to 90 °

  Obtuse – Between 90° and 180°

  Straight – Equal to 180°

- Angle Pairs

  Complementary - Two angles that add up to 90°

  Supplementary - Two angles that add up to 180°

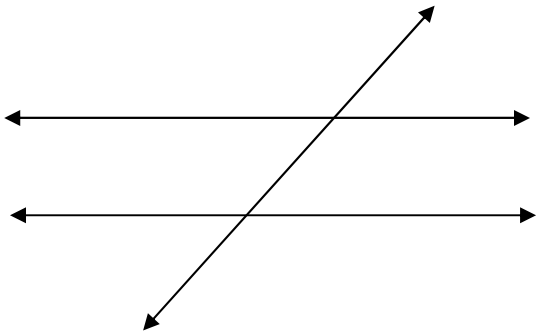
  Vertical – Equal angles that are formed by making an X

Triangles

- The angles of a triangle add up to 180°.
- The measure of an exterior angle of a triangle is equal to the sum of the two interior angles.
- Special Lines for Triangles:
  - Median – A line drawn from a vertex to the midpoint of the opposite side
  - Altitude – A line drawn from the vertex perpendicular to the opposite side.
  - Midsegment – A line connecting the midpoints of two sides; parallel and ½ as long as 3rd side
- The length of each side of a triangle must be less than the sum of the other two sides.
- A line drawn parallel to one side of a triangle and intersecting the other two sides divides these sides into proportional segments.
- A scalene triangle has three different length sides and three different size angles.
  - A triangle can have at the most one obtuse angle.
  - The longest side is opposite the widest angle.
- An isosceles triangle has two equal sides and two equal angles opposite them
- An equilateral triangle has three equal sides and all angles equal 60°.
- A right triangle has one 90° angle.
  - The sum of the square of the legs of a right triangle is equal to the square of the hypotenuse. $a^2 + b^2 = c^2$
  - A Pythagorean triple is a set of three numbers that can be the lengths of the sides of a right triangle, such as {3, 4, 5}, {5, 12, 13} and {8, 15, 17}.
  - In a 30-60-90 triangle, the hypotenuse is twice as long as the short side, and the middle side is $\sqrt{3}$ as long as the short side
  - In a 45-45-90 triangle, the hypotenuse is $\sqrt{2}$ times as long as either leg.
  - The altitude to the hypotenuse divides the triangle into two smaller similar triangles that are proportional to the original triangle.

Parallel Lines

- Parallel lines have the same slope and never meet.
- When parallel lines are crossed by a transversal,
  the alternate interior angles are congruent,
  the alternate exterior angles are congruent,
  the vertical angles are congruent,
  the corresponding angles are congruent
  the same side interior angles are supplementary.



Parallelograms

- A parallelogram has two pairs of parallel sides.
- The opposite sides of a parallelogram are congruent.
- The opposite angles of a parallelogram are congruent.
- Adjacent angles in a parallelogram are supplementary.
- The diagonals of a parallelogram bisect each other.

Special Parallelograms

- A rectangle is a parallelogram with four congruent angles, each equal to 90°.
- The diagonals of a rectangle are congruent.
- A rhombus is a parallelogram with four congruent sides.
- The diagonals of a rhombus bisect the corner angles.
- The diagonals of a rhombus are perpendicular to each other.
- A square is a rectangle and a rhombus.

Trapezoids

- The median of a trapezoid is a line drawn from the midpoint of one leg to the midpoint of the other. The median is parallel to both bases and its length is ½ the sum of the bases.
- An isosceles trapezoid has congruent legs.
  - The diagonals of an isosceles trapezoid are congruent.
  - Each pair of base angles in an isosceles trapezoid is congruent.

Polygons

- The sum of the interior angles of a polygon having N sides is 180°(N-2).
- The sum of the exterior angles (one at each vertex) of any polygon is 360°.
- For regular polygons, an exterior angle equals 360 divided by the number of sides.
- For regular polygons, an interior angle equals 180 minus an exterior angle.
- For a regular polygon, the central angles are congruent and equal to 360 divided by the number of sides.

Circles

- In a circle, congruent chords have congruent arcs (and congruent arcs have congruent chords).
- In a circle, congruent chords are equidistant from the center.
- A radius drawn perpendicular to a chord always bisects the chord and the intercepted arc.
- A radius drawn to a point of tangency is perpendicular to the tangent line.
- A central angle is an angle with its vertex at the center of the circle.
- The measure of a central angle is equal to the measure of the arc it intercepts.
- An inscribed angle is an angle with its vertex on the circle.
- The measure of an inscribed angle is equal to ½ the measure of the arc it intercepts.
- Inscribed angles that intercept the same arc are congruent. If inscribed angles are congruent, their intercepted arcs are congruent.
- The angle formed by two intersecting chords is equal to the average of the intercepted arcs.
- The angle formed by two intersecting secants is equal to ½ (the big intersected arc minus the smaller arc).
- A tangent touches a circle at only one point.
- If two tangent segments are drawn to a circle from the same point, they are congruent.

Perimeter

- The circumference of a circle is $C = \pi$ x Diameter.
- The length of an arc is L = Circumference of circle x Degree measure of arc/360.
- If two polygons are similar, the ratio of their perimeters is equal to the ratio of the lengths of their sides.

Area

- The area of a rectangle is A = Base x Height
- The area of a triangle is A = ½ (Base x Height)
- The area of a parallelogram is A = Base x Height (the height must be measure perpendicular to the base)
- The area of a rhombus can be calculated using its diagonals, $A = \frac{1}{2}(Diagonal_1 \times Diagonal_2)$
- Area of a Trapezoid = $\frac{1}{2} \times (Base_1 + Base_2) \times Height$
- The area of a circle is $A = \pi r^2$, where r is the radius of the circle.
- The area of a sector is S = Area of circle x Degree measure of arc/360.
- If two polygons are similar, the ratio of their areas is equal to the ratio of the length of their sides squared.

Volume

- The volume of a prism is equal to V= Area of base x Height of solid.
- The volume of a cone or pyramid is V = 1/3 (Area of base x Height).
- The volume of a sphere is $V = 4/3\ \pi r^3$.
- The ratio of the volumes of similar solids is equal to the ratio between the sizes of the shapes cubed.

Cartesian Coordinates

- The distance between two points is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
- To find the midpoint between two points, just average the x-coordinates and then the y-coordinates.
- The slope of a line equals its rise ↑ divided by its run →.
- A horizontal line has a slope of zero.
- A vertical line has undefined slope.
- A line with a positive slope goes uphill from left to right.
- A line with a negative slope goes downhill from left to right.
- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals.

LINES

In the diagram, points V, W, X, Y, and Z are collinear. VZ = 52, XZ = 20, and WX=XY= YZ. Find the indicated lengths.

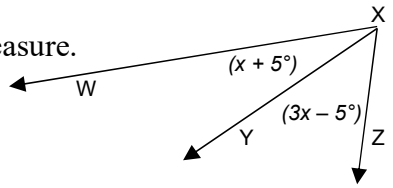
1. WX
2. VW
3. WY
4. VX
5. WZ
6. VY

7. Point S is between R and T on RT. Use the given information to write an equation in terms of x. Solve the equation. RS = 2x + 10 ST = x – 4 RT = 21 Find the lengths of RS and ST.

ANGLES

Use the given information to find the indicated angle measure.

8. Given m∠WXZ = 80°, find m∠YXZ.



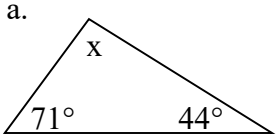
9. Angle A and Angle B are complements. The measure of Angle A is four times as big as the measure of Angle B. Find the measure of both angles.

10. Angle C and Angle D are supplements. Angle C = 3x – 5. Angle B = 2x + 25. Find the measure of both angles.

TRIANGLES

1. Find the measure of the missing angle x.

a.



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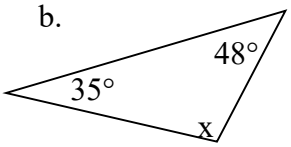

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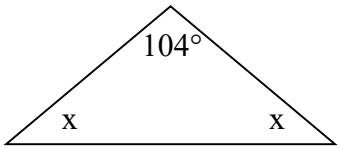
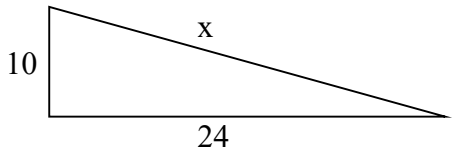


Use Pythagorean Triples to solve for the missing lengths.

2.



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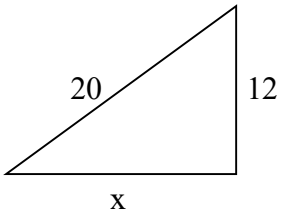

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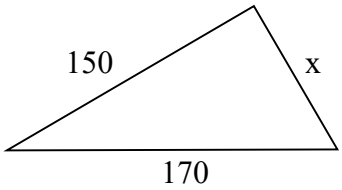

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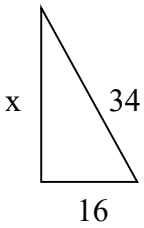


6. Use the distance formula or Pythagorean Theorem to calculate the distance between the following pairs of points:
   a. (2, 8) and (7, -4)
   b. (5, 3) and (-1, -9)

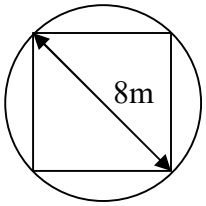
Use the Pythagorean Theorem and Pythagorean Triples to solve the word problems.

7. A rectangle is 9 inches long and 12 inches wide. How long is its diagonal?

8. The perimeter of a square is 24 feet, how long is the diagonal of the square?

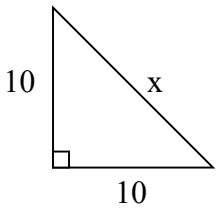
9. Sam drives 24 miles north, then turns and drives 45 miles east. How far is Sam from where she started?

10. A circle has a diameter of 8 meters. How long are the sides of the largest square you can draw inside the circle? What is the area of the square?



These are "special" triangles. Solve for the missing values for each triangle.

1.



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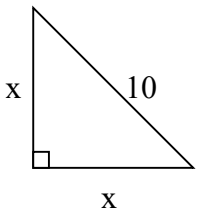

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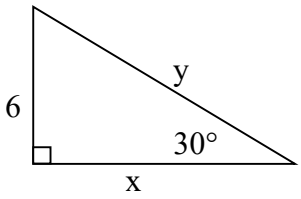

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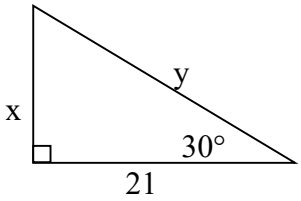

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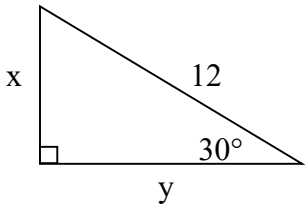

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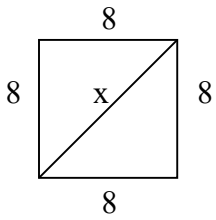

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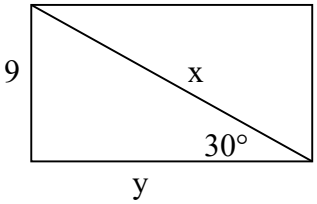

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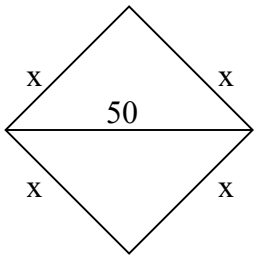
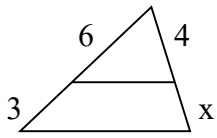
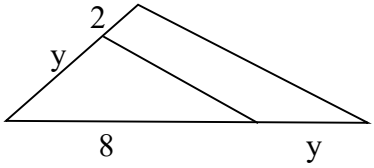


Use proportions to solve for the missing value.

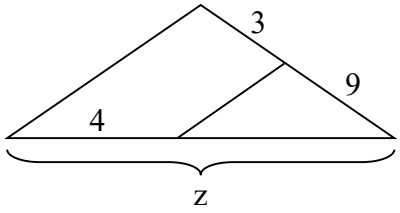
9. x =



10. y =

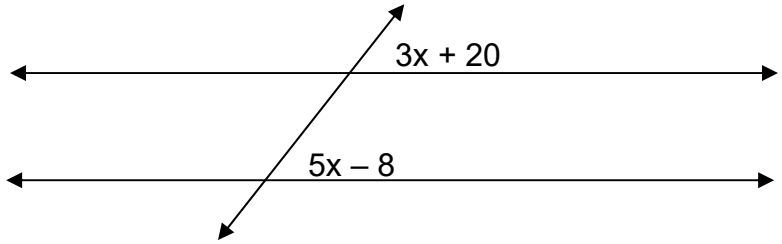


11. z =

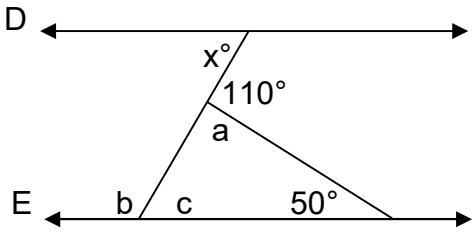


PARALLEL LINES

1. Lines A and B are parallel. Find x.



2. Line D is parallel to line E. Find the measures of a, b, c, and x.



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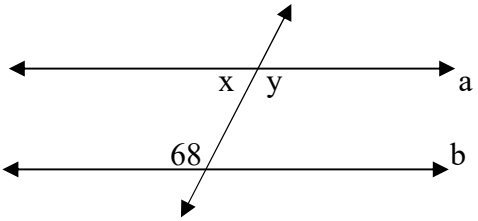

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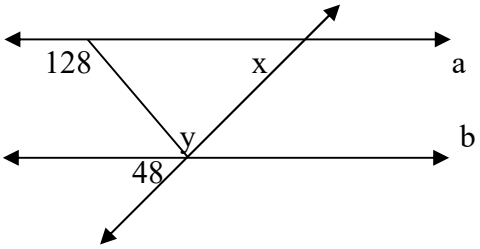
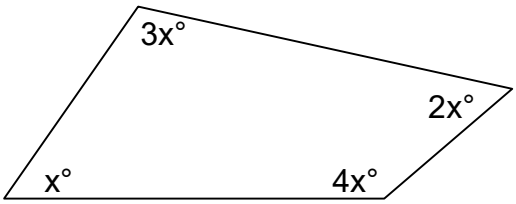


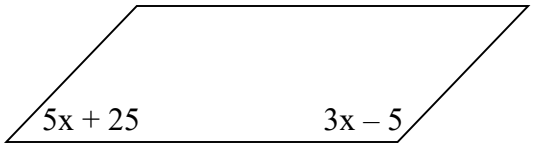
QUADRILATERALS & POLYGONS

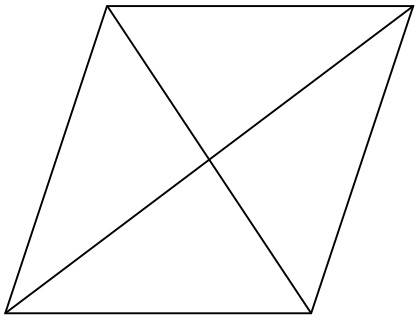
1. The height of a rectangle is 6 feet and the length of one diagonal is 12 feet. Find the perimeter of the rectangle.

2. For the quadrilateral shown below, find x.



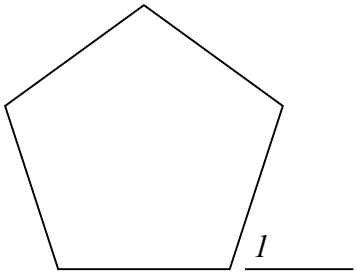
This is a parallelogram.
Solve for x.

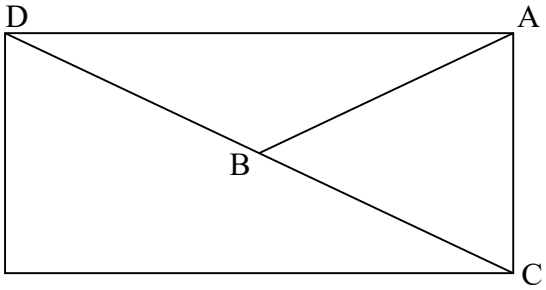




This shape is a rhombus. The long diagonal is 24 cm. The short diagonal is 10 cm. How long is each side?

This is a regular pentagon.
What is the measure of ∠1?

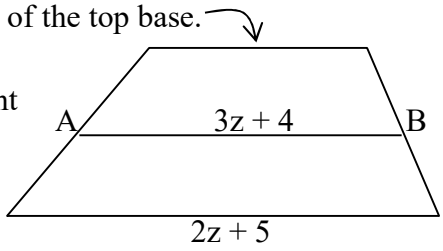




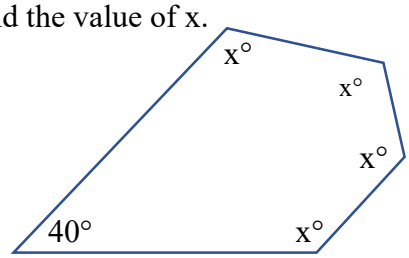
B is the midpoint of the diagonal.
AB = 13, AC = 10. What is the length of AD?

Write an expression that equals the length of the top base.

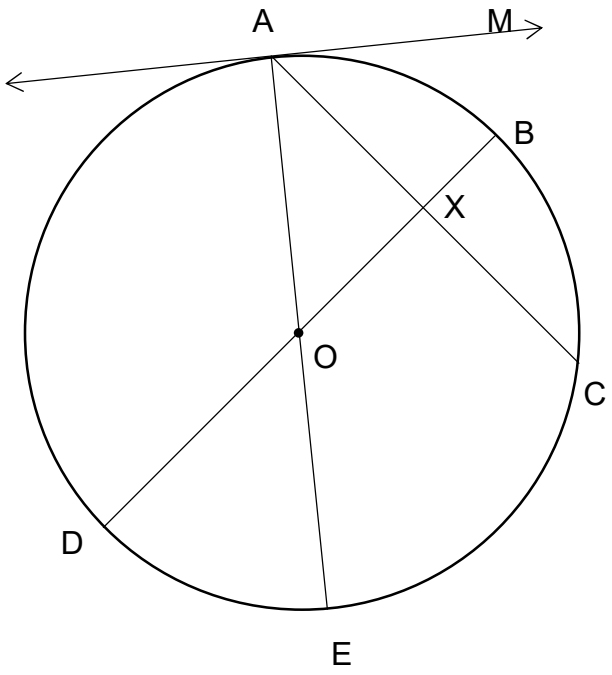
AB is the midsegment



Find the value of x.



CIRCLES:

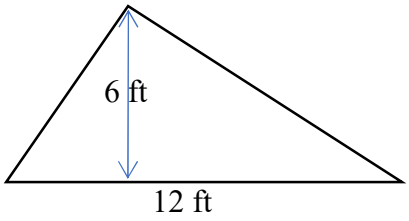


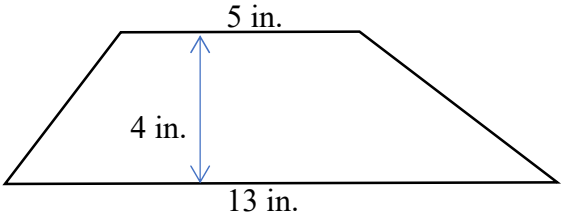
O is the center of the circle.
AE = 20, AX= 8, ∠OAX = 36°
DB bisects AC. AM is tangent.
Find the following measures.

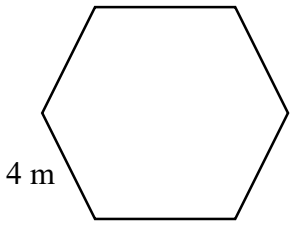
1. m∠OXA
2. $\overline{XC}$
3. $\overline{OX}$
4. $\overline{XB}$
5. m $\overset{\frown}{EC}$
6. m $\overset{\frown}{ABC}$
7. m $\overset{\frown}{AB}$
8. m∠MAE
9. m $\overset{\frown}{AD}$
10. m $\overset{\frown}{BC}$

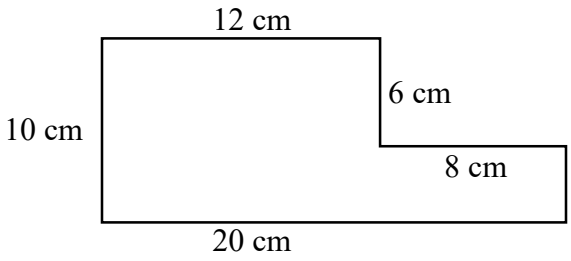
11. A circle has a radius of 15 feet. Find its:
    a. Circumference
    b. Arc length of 1/3 of the circle
    c. Arc length of 40° of the circle
    d. Arc length of $\frac{\pi}{5}$ of the circle
    e. Area
    f. Area of 2/3 of the circle
    g. Area of 48° of the circle.

AREA









1. The volume of a rectangular prism that is 2 cm wide, 5 cm tall, and 6.4 cm long is equal to the volume of a cube. How long are the sides of the cube?

2. Rectangle A is similar to Rectangle B. The area of Rectangle A is 10 in^2 and the area of Rectangle B is 40 in^2. If the base of Rectangle A is 7 inches, how long is the base of Rectangle B?

CARTESIAN COORDINATES

1. Find the midpoint between (-4, 1) and (8, 11).

2. M is the midpoint of segment AB. M is at (2, 5). B is at (8, 11). What is the coordinate of A?

3. Find the slope of a line parallel to $3x + 2y = 15$.

4. What is the slope of a line perpendicular to $y = \frac{3}{4}x - 7$?

5. Find the slope of a line perpendicular to the line through (2, 3) and (4, -1).