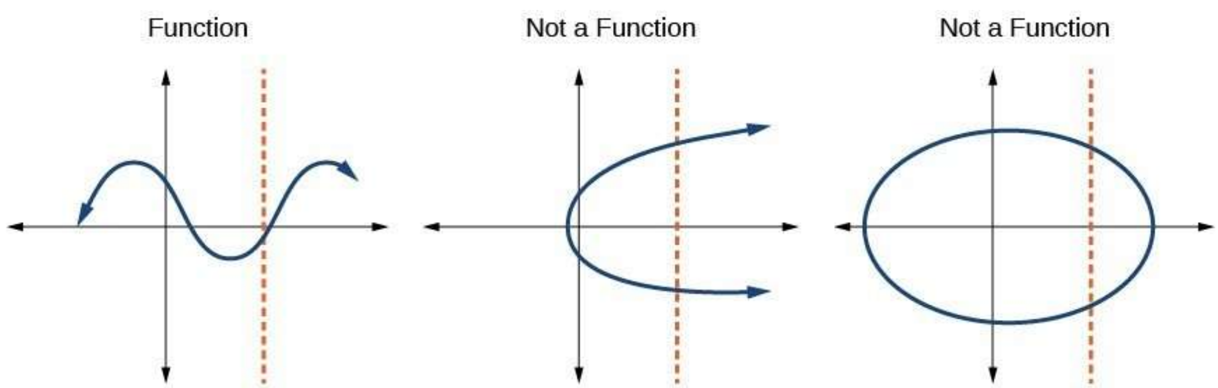
Chapter 2: Functions

2.1 What is a Function?

In math, a function is a rule that pairs each input to exactly one output.

Functions can be represent several ways. One circle diagram below shows a function and the other does not.

Functions can be graphed. To determine if a graph is a function, use the **Vertical Line Test**. Draw a vertical line anywhere on the graph. If a vertical line crosses a curve more than one time, it is not a function. If every possible vertical line crosses the curve at only one point (or not at all), the graph is a function.



2.2 Domain and Range

The domain of a function is its input. This is the set of values that can be put into the function. The letter that represents the input is called the independent variable because you can change it and see its effect.

The range of the function is the output, or the dependent variable. This is the set of values that the functions produces when the values of the domain are used in the function.

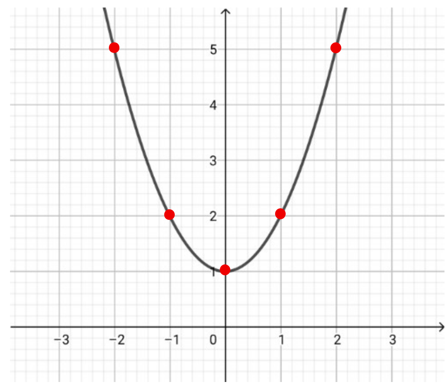
The horizontal axis represents the domain of the function, and the vertical axis represents the range.

2.3 Graphing Functions

To create a graph of a function, make a table of x and y values. Substitute each value of x into the function and calculate its y-value. This creates a table of x and y coordinates. Plot the coordinates on a grid and connect the points with a smooth line or curve.

Example: $y = x^2 + 1$

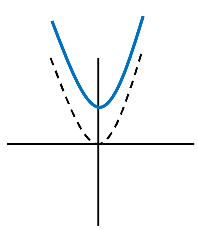
x	y	
-2	$(-2)^2 + 1 = 5$	(-2, 5)
-1	$(-1)^2 + 1 = 2$	(-1, 2)
0	$0^2 + 1 = 1$	(0, 1)
1	$1^2 + 1 = 2$	(1, 2)
2	$2^2 + 1 = 5$	(2, 5)



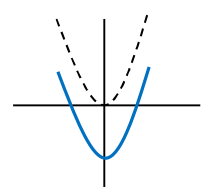
2.4 Transformations

When the shape of a function stays the same, but it moved up, down, left, right, or "flipped over, it's called a transformation.

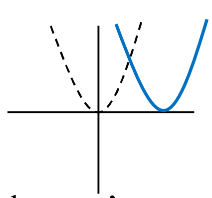
Vertical shift c units **up**:
Add a value to the function
$y = f(x) + c$



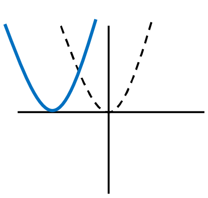
Vertical shift c units **down**:
Subtract a value from the function
$y = f(x) - c$



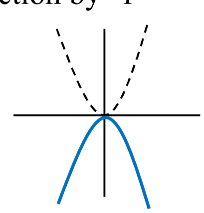
Horizontal shift c units to the **right**:
Subtract a value from x
$y = f(x-c)$



Horizontal shift c units to the **left**:
Add a value to x
$y = f(x+c)$

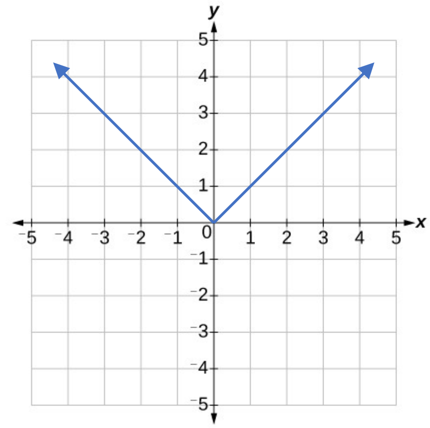


Reflection about the **x-axis**:
Multiply the function by -1
$y = -f(x)$

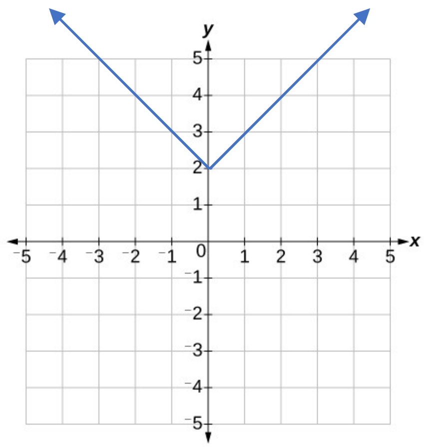


Example: For the function shown here, several transformations of the function are shown below.

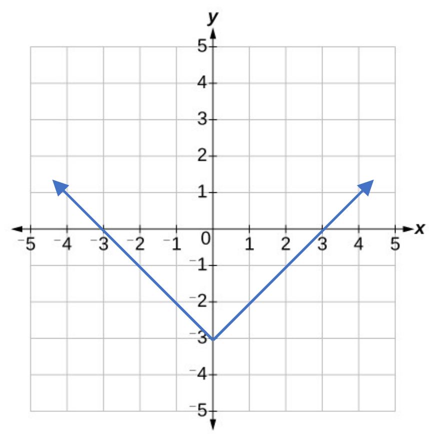
$y = |x|$



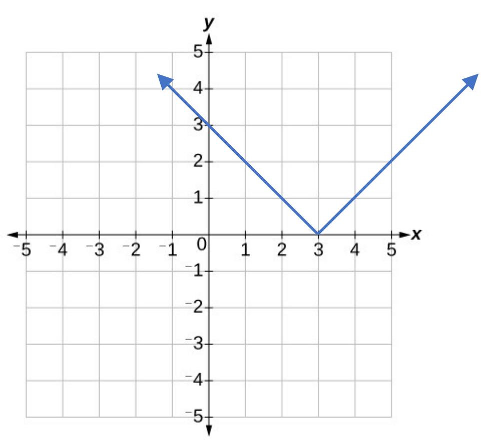
$y = |x| + 2$



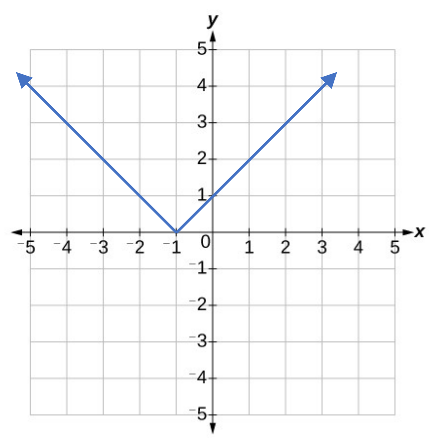
$y = |x| - 3$



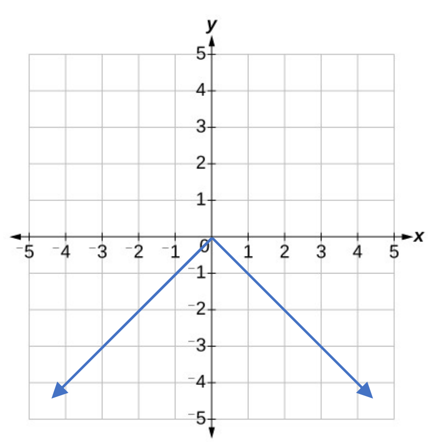
$y = |x - 3|$



$y = |x + 1|$



$y = -|x|$

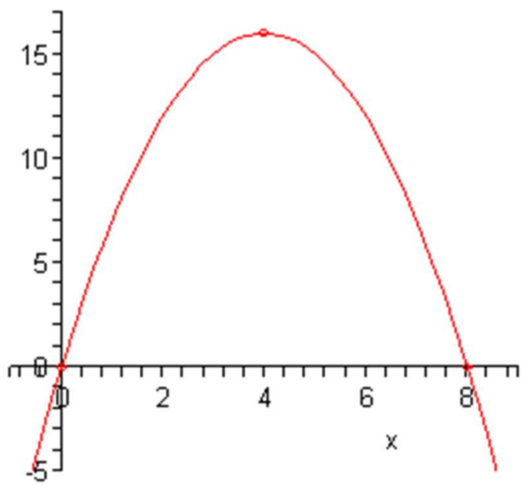


2.5 Real-World Functions

They say “a picture is worth a thousand word”. Sometimes it’s easier to understand things if we see a picture or diagram of it.

Real-World Functions

Functions of real-world events come in all shapes. Their domains and ranges have to reflect data that can actually occur. When you kick a football, the path of the ball can be represented by a parabola. The graph draw below could represent the path of the football, but would the football go underground? The graph continues below the x-axis. If this modeled a real football, that would be telling you that the ball traveled underground. Also, the ball is below the ground before it gets kicked!



To make the graph represent real events, we would set its domain to [0, 8]. This would limit the range to [0, 16].

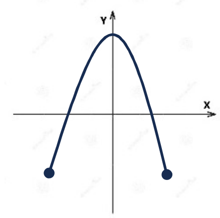
The variable on the x-axis is the independent variable. It is the “input” into the function and it affects the “output”.

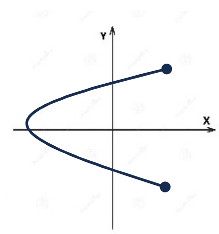
The variable on the y-axis is the dependent variable. Its value depends on what the x-value is.

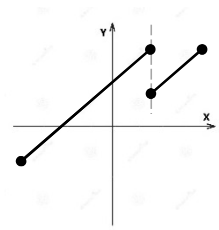
PRACTICE

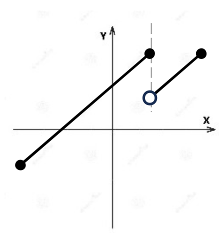
Section 2.1

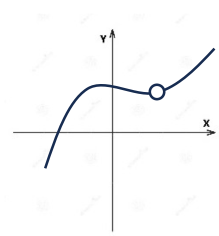
Tell whether each picture shows a function or not.

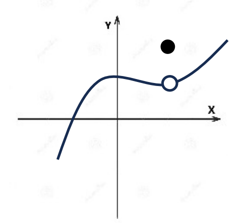
1. 

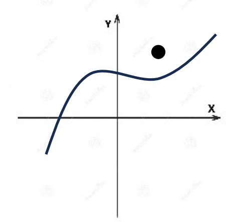
2. 

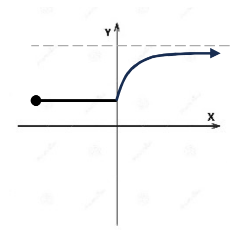
3. 

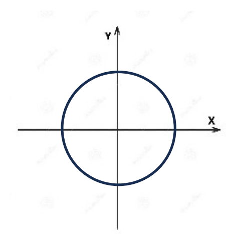
4. 

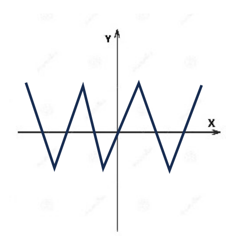
5. 

6. 

7. 

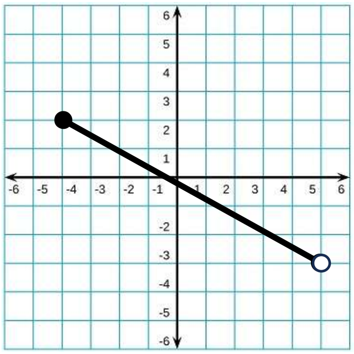
8. 

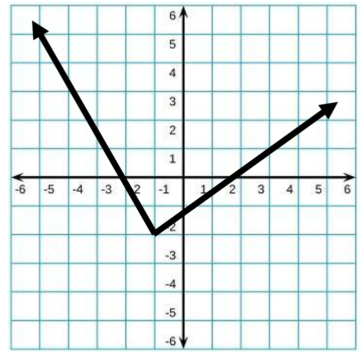
9. 

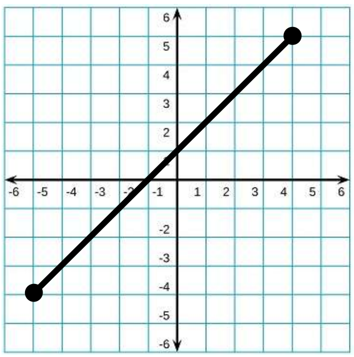
10. 

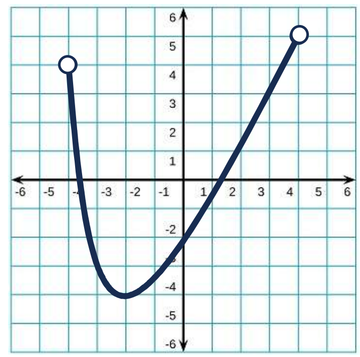
Section 2.2

Tell the domain and range of each function. Use interval notation.

11. 

13. 

12. 

14. 

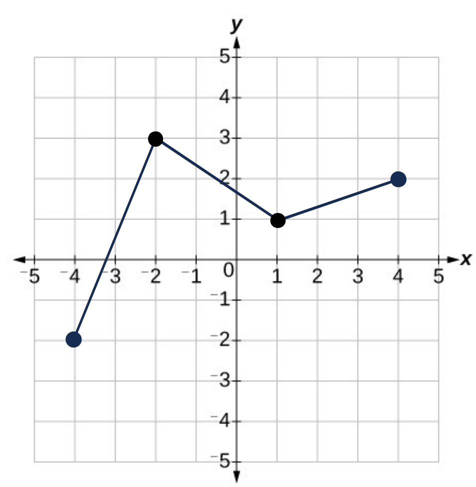
Section 2.3

Create a table of values for each function. Your values must be within the given domain. On graph paper, plot an accurate graph of the function.

15. $y = \frac{1}{2}x - 4$; domain=[-2, 6]
16. $y = 2x + 3$: domain = (-2, 4]
17. $y = -|x - 2|$; domain = (-3, 3)
18. $y = x^2 - 4x$; domain = {all real numbers}

Section 2.4

The function f(x) is drawn on the grid to the right.
On graph paper, accurately plot points and draw each transformation.



19. $y = f(x) + 2$
20. $y = f(x) - 1$
21. $y = f(x + 4)$
22. $y = f(x - 3)$
23. $y = -f(x)$
24. $y = f(x - 2) + 1$

Section 2.5

Sketch a reasonable graph with a reasonable domain for each function described. Label the x-axis and the y-axis.

1. The weight of an average person depends on his or her height.
2. The height of a pop fly depends on the number of seconds since the batter hit it.
3. You fill up your car with 15 gallons of gas. The amount of gas in your tank depends on how far you have driven.
4. You pull the stopper out of the bathtub to let the water drain out. The amount of water in the tub depends on the number of seconds since you pulled the stopper.
5. You climb to the top of Wursthalle and drop a golf ball. The distance the ball is above the ground depends on the number of seconds since you've dropped it. (Wursthalle is 40 feet tall. Do NOT try to measure it.)
6. You take an empty balloon and blow it up with your breath. The diameter of the balloon is related to the number of breaths you have blown into it.
7. You stop at On the Grind before class to get a coffee, but you left it in your car. The temperature of your coffee depends on how long it has been sitting in your car.
8. You dive off the 3-meter diving board at Landa Park Pool. Your position in relation to the water's surface is related to the number of seconds since you dived.
9. Your efficiency at studying Algebra 2 depends on how late it is at night.