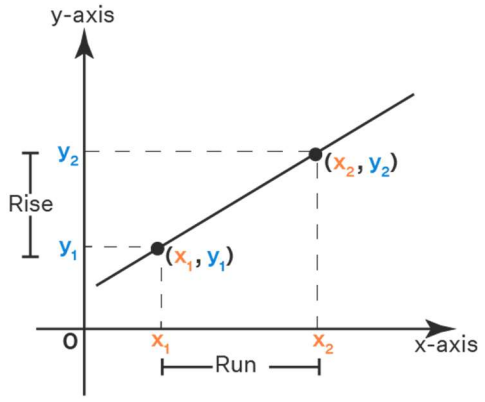
Chapter 3: Lines

Slope

The slope of a line is the rise divided by the run.

$$\text{Slope} = \frac{Rise \uparrow\downarrow}{Run \rightarrow} = \frac{y_2 - y_1}{x_2 - x_1}$$



Slope is sometimes called the **rate of change**.

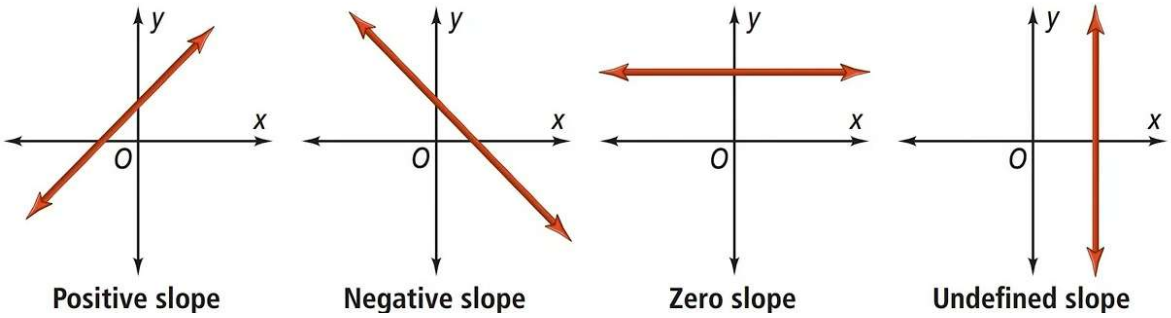
Types of Slope

Positive slope goes "uphill".

Negative slope goes "downhill"

Zero slope is horizontal.

Undefined slope is vertical.



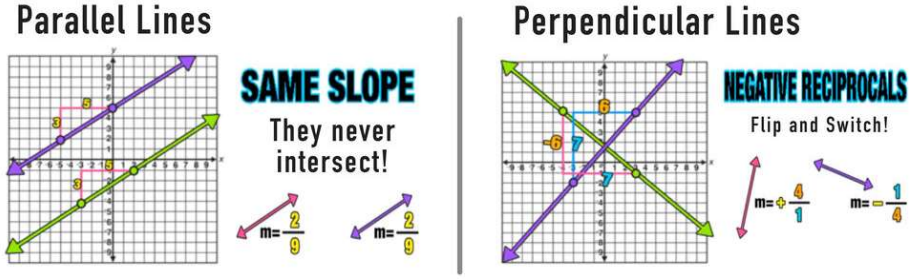
Parallel and Perpendicular Slope

If two lines are parallel, they have the same slope.

If two lines are perpendicular, their slopes are negative reciprocals.

Slope = 4 Perpendicular slope = $\frac{-1}{4}$

Slope = $\frac{-2}{3}$ Perpendicular slope = $\frac{3}{2}$



Graphing Lines

Slope-Intercept Method

Use this method if the line is in slope-intercept form, $y = mx + b$.

1. Put a small dot where the y-intercept is on the y-axis.
2. Using the slope, count up (or down if the slope is negative), then over to the right, and place another small dot.
3. Repeat the pattern of up and over (or down and over if the slope is negative) until you see a straight line of dots.
4. Connect the dots with a straight-edge.

Using Intercepts

Use this method if the line is in standard form, $Ax + By = C$.

1. Cover up the By in the formula. This gives you a smaller equation, $Ax = C$. Solve for x.
2. Put a small dot on the x-axis at the point you just found.
3. Cover up the Ax in the formula. This gives you a smaller equation, $By = C$. Solve for y.
4. Put a small dot on the y-axis at the point you just found.
5. Connect the dots with a straight-edge.

Three Forms of Lines

Name	Formula	Example	Features
Slope-Intercept	$y = mx + b$	$y = \frac{1}{2} x + 7$	slope = ½ Y-intercept at (0, 7)
Point-Slope	$y - y_1 = m(x - x_1)$	$y - 3 = -2(x - 4)$	slope = -2 Contains point (4, 3)
Standard Form	$Ax + By = C$	$3x + 8y = 12$	

Changing Forms

Point-Slope to Slope-Intercept

1. Distribute the slope to get rid of the parentheses.
2. Add or subtract y_1 to get the y by itself.

Slope-Intercept to Standard Form

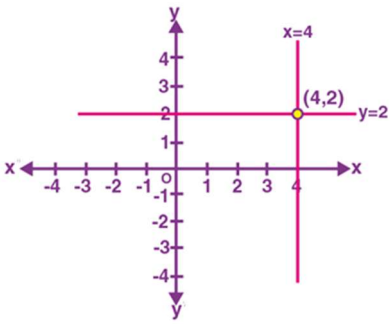
1. Add or subtract the mx term from both sides.
2. If there are fractions in the equation, multiply the entire equation by the common denominator to get rid of fractions.
3. If the coefficient in front of x is negative, multiply the entire equation by -1.

Standard Form to Slope-Intercept

1. Subtract the x-term to get it on the right side of the equation.
2. Divide by the coefficient in front of the y.

Horizontal and Vertical Lines

Horizontal	y = *number*	y = 2	The line crosses at (0, 2)
Vertical	x = *number*	x = 4	The line crosses at (4, 0)

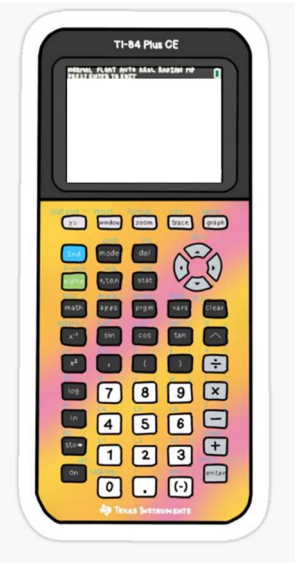


Writing Equations of Lines

- If they give you the slope or rate of change, write an equation using the slope-intercept or point-slope form.
- If they give you the coordinates of two points, calculate the slope between the points and then use the point-slope form.
- If the line is horizontal or the rate of change is zero, it's a horizontal line. y = *number*
- If the line has undefined slope, it's a vertical line. x = *number*
- For word problems, try drawing a graph using the data you are given, then write the equation of the line.

Linear Regression

Linear regression is a method for determining the best-fit line through a set of data points. You will use the STAT function on your calculator to find the equation of a line.



1. Clear all lists in your calculator's memory.
   Press 2nd, MEM, 4.

2. Enter the x-values into List 1.
   Press STAT, 1. Move the cursor over to L1 and type in the numbers.

3. Enter the y-values into List 2.
   Move the cursor over to L2 and type in the numbers.

4. Calculate the linear equation.
   Press STAT, CALC, 4.
   Press ENTER until the calculator solves the problem.
   Write down the values for a and b that are displayed.
   The slope of your line is "a" and the y-intercept is "b".

6. Enter the equation of the line in your calculator so you can see the graph.

PRACTICE

Determine the slope between the two points.

1. (2, 3), (4, 0)
2. (-2, 3), (2, 1)
3. (-3, -1), (5, -1)
4. (2, -8), (2, 4)
5. (-5, -4), (-1, 3)

Calculate the slope perpendicular to the given slope.

6. m = 2/5
7. m = -1/2
8. m = 7
9. m = -3
10. m = 0

Determine whether the lines are parallel, perpendicular, or neither.

11. Line A: through (3, - 1) and (6, -4)
    Line B: through (-4, 5) and (-2, 7)

12. Line A: through (1, 5) and (3, -2)
    Line B: through (-3, 2) and (4,0)

On graph paper, accurately graph the line using the slope and y-intercept.

13. $y = \frac{3}{4}x + 1$
14. $y = -2x + 6$
15. $y = x - 3$
16. $y = -\frac{1}{2}x$

On graph paper, accurately graph the line using x and y-intercepts.

17. $4x + 2x = 12$
18. $3x - y = 6$
19. $2x - 5y = 10$

On graph paper, accurately graph the horizontal and vertical lines.

20. $y = 5$
21. $y = -2$
22. $x = 4$
23. $x = -1$

Change the equation to (a) slope-intercept form, and (b) general form.

24. $y - 7 = 3(x - 1)$
25. $y + 1 = \frac{3}{4}(x - 4)$
26. $y - 2 = -\frac{1}{2}(x + 6)$
27. $y + 3 = \frac{2}{3}(x - 5)$
28. $y - 8 = -\frac{1}{4}(x - 7)$

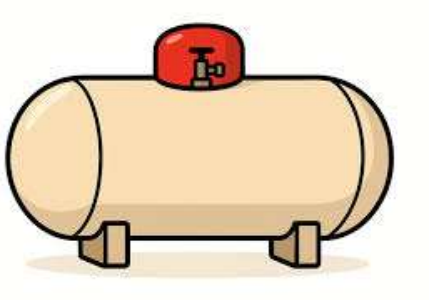
Use either the slope intercept $(y=mx+b)$ or point slope $(y-y_1=m(x-x_1))$ formula for each line. Horizontal and vertical lines can be written as $y = number$ or $x = number$.

29. Slope = 4; y-intercept = -6
30. Slope = ¼; y-intercept = 5
31. Slope =2; point on line (7, -3)
32. Slope = -2/3; point on line (6.-4)
33. Two points on the line (-4, 6) (2, 1)
34. Two points on the line (8, 5) (3, -4)
35. x-intercept (4,0); y-intercept (0,-5)
36. Parallel to y = 6x – 11; contains point (-5, 2)
37. Parallel to $y = {}^{2}/_{5}x + 6$; contains point (8, 11)
38. Perpendicular to y = 2x – 7; contains point (-3, 1)
39. Perpendicular to $y = -{}^{4}/_{3}x - 8$; contains point (-2, 7)
40. Horizontal line through point (-9, 14)
41. Horizontal line through (4, -8)
42. Vertical line through (63 10)
43. Vertical line through (-7, 11)

44. A bakery sells a box of 10 cookies for $15 and a box of 30 cookies for $40.
    a. Write the linear equation that relates the number of cookies (x) to the price (y).
    b. Accurately graph this equation on graph paper. Use appropriate scales. Label both axes.
    c. Using your equation from part a, determine the price for a box of 100 cookies.
    d. How many cookies would be in a box that cost $95?

45. Andrea has $450 in her savings account. Each week she will take $80 from her paycheck and add it to her savings.
    a. Write the linear equation that relates the weeks (x) to the amount in her savings (y).
    b. Accurately graph this equation on graph paper. Use appropriate scales. Label both axes.
    c. Using your equation from part a, determine how much money she will have in her savings after one year (52 weeks).
    d. When she gets $8000 in her account, she's going to take a trip to Europe. How many weeks will it take her to accumulate $8000? (Round up to the nearest week. Do not round down.)

46. Over a 30 day period, the amount of propane in a tank that stores propane for a home decreases from 400 gallons to 214 gallons.



   a. What is the average rate of change of the amount of propane?
   b. Write the linear equation that relates the days (x) to the gallons of propane (y).
   c. How much propane would be in the tank after 40 days?
   d. How long would it take to use up all the propane at this rate? (Round up to the nearest day. Do not round down.)

47. A web hosting service charges an initial $150 setup charge and $15 per month for hosting.
   a. Write the linear equation that relates the number of months (x) to the cost (y).
   b. If you use this service for 5 years (60 months), how much would it cost?

48. Ace Plumbing Company charged $220 for a job that took 2 hours and $480 for a job that took 6 hours.

   a. Use this information to write the equation of the line relating hours (x) to cost (y).
   b. How much does Ace Plumbing charge per hour?
   c. What is the flat fee for each job?
   d. If a job took 4.5 hours to complete, how much would it cost?

Use the Linear Regression function of your calculator to find the best-fit line through these points. Round numbers to two decimal places.

49. (2, 5); (6, 11); (10, 15)
50. (-3, 12); (1, 7); (4, -4)
51. (-.4, 0.5); (2, 2); (6, 3); (8, 3.5)