

MATCHED PROBLEM 6

Find

$$(A) \frac{d}{dx} \ln(x^3 + 2x) \quad (B) \frac{d}{dx} e^{3x^2+2} \quad (C) \frac{d}{dx} (2 + e^{-x^2})^4$$

Answers to Matched Problems

- $f[g(x)] = 2e^x$, $g[f(u)] = e^{2u}$
- (A) $f(u) = 50e^u$, $u = -2x$ (B) $f(u) = \sqrt[3]{u}$, $u = 1 + x^3$
[Note: There are other correct answers.]
- (A) $15(5x + 2)^2$
(B) $20x^3(x^4 - 5)^4$
(C) $-4t/(t^2 + 4)^3$
(D) $-1/(2\sqrt{4 - w})$
- (A) $\frac{dy}{dx} = -5u^{-4}$, $\frac{du}{dx} = 6x^2$, $\frac{dy}{dx} = -30x^2(2x^3 + 4)^{-6}$
(B) $\frac{dy}{du} = e^u$, $\frac{du}{dx} = 12x^3$, $\frac{dy}{dx} = 12x^3 e^{3x^4+6}$
(C) $\frac{dy}{du} = \frac{1}{u}$, $\frac{du}{dx} = 2x + 9$, $\frac{dy}{dx} = \frac{2x + 9}{x^2 + 9x + 4}$
- $\frac{3e^x[\ln(1 + e^x)]^2}{1 + e^x}$
- (A) $\frac{3x^2 + 2}{x^3 + 2x}$ (B) $6xe^{3x^2+2}$ (C) $-8xe^{-x^2}(2 + e^{-x^2})^3$

Exercise 11-4

For many of the problems in this exercise set, the answers in the back of the book include both an unsimplified form and a simplified form. When checking your work, first check that you applied the rules correctly, and then check that you performed the algebraic simplification correctly.

A In Problems 1–4, find $f[g(x)]$.

- $f(u) = u^3$; $g(x) = 3x^2 + 2$
- $f(u) = u^4$; $g(x) = 1 - 4x^3$
- $f(u) = e^u$; $g(x) = -x^2$
- $g(u) = e^u$; $g(x) = 3x^3$

In Problems 5–8, write each composite function in the form $y = f(u)$ and $u = g(x)$.

- $y = (3x^2 - x + 5)^4$
- $y = (2x^3 + x + 3)^5$
- $y = e^{1+x+x^2}$
- $y = e^{x^4+2x^2+5}$

In Problems 9–16, replace the ? with an expression that will make the indicated equation valid.

- $\frac{d}{dx} (3x + 4)^4 = 4(3x + 4)^3$?
- $\frac{d}{dx} (5 - 2x)^6 = 6(5 - 2x)^5$?
- $\frac{d}{dx} (4 - 2x^2)^3 = 3(4 - 2x^2)^2$?
- $\frac{d}{dx} (3x^2 + 7)^5 = 5(3x^2 + 7)^4$?

- $\frac{d}{dx} e^{x^2+1} = e^{x^2+1}$?
- $\frac{d}{dx} e^{4x-2} = e^{4x-2}$?
- $\frac{d}{dx} \ln(x^4 + 1) = \frac{1}{x^4 + 1}$?
- $\frac{d}{dx} \ln(x - x^3) = \frac{1}{x - x^3}$?

In Problems 17–36, find $f'(x)$ and simplify.

- $f(x) = (2x + 5)^3$
- $f(x) = (3x - 7)^5$
- $f(x) = (5 - 2x)^4$
- $f(x) = (9 - 5x)^2$
- $f(x) = (4 + 0.2x)^5$
- $f(x) = (6 - 0.5x)^4$
- $f(x) = (3x^2 + 5)^5$
- $f(x) = (5x^2 - 3)^6$
- $f(x) = e^{5x}$
- $f(x) = 6e^{-2x}$
- $f(x) = 3e^{-6x}$
- $f(x) = e^{x^2+3x+1}$
- $f(x) = (2x - 5)^{1/2}$
- $f(x) = (4x + 3)^{1/2}$
- $f(x) = (x^4 + 1)^{-2}$
- $f(x) = (x^5 + 2)^{-3}$
- $f(x) = 3 \ln(1 + x^2)$
- $f(x) = 2 \ln(x^2 - 3x + 4)$
- $f(x) = (1 + \ln x)^3$
- $f(x) = (x - 2 \ln x)^4$

In Problems 37–42, find $f'(x)$ and the equation of the line tangent to the graph of f at the indicated value of x . Find the value(s) of x where the tangent line is horizontal.

- $f(x) = (2x - 1)^3$; $x = 1$
- $f(x) = (3x - 1)^4$; $x = 1$

39. $f(x) = (4x - 3)^{1/2}; x = 3$

40. $f(x) = (2x + 8)^{1/2}; x = 4$

41. $f(x) = 5e^{x^2-4x+1}; x = 0$

42. $f(x) = \ln(1 - x^2 + 2x^4); x = 1$

In Problems 43–62, find the indicated derivative and simplify.

43. y' if $y = 3(x^2 - 2)^4$

44. y' if $y = 2(x^3 + 6)^5$

45. $\frac{d}{dt} 2(t^2 + 3t)^{-3}$

46. $\frac{d}{dt} 3(t^3 + t^2)^{-2}$

47. $\frac{dh}{dw}$ if $h(w) = \sqrt{w^2 + 8}$

48. $\frac{dg}{dw}$ if $g(w) = \sqrt[3]{3w - 7}$

49. $g'(x)$ if $g(x) = 4xe^{3x}$

50. $h'(x)$ if $h(x) = \frac{e^{2x}}{x^2 + 9}$

51. $\frac{d}{dx} \frac{\ln(1+x)}{x^3}$

52. $\frac{d}{dx} [x^4 \ln(1+x^4)]$

53. $F'(t)$ if $F(t) = (e^{t^2+1})^3$

54. $G'(t)$ if $G(t) = (1 - e^{2t})^2$

55. y' if $y = \ln(x^2 + 3)^{3/2}$

56. y' if $y = [\ln(x^2 + 3)]^{3/2}$

57. $\frac{d}{dw} \frac{1}{(w^3 + 4)^5}$

58. $\frac{d}{dw} \frac{1}{(w^2 - 2)^6}$

59. $\frac{dy}{dx}$ if $y = (3\sqrt{x} - 1)^5$

60. $\frac{dy}{dx}$ if $y = \left(\frac{1}{x^2} - 5\right)^{-2}$

61. $f'(t)$ if $f(t) = \frac{4}{\sqrt{t^2 - 3t}}$

62. $g'(t)$ if $g(t) = \frac{3}{\sqrt[3]{t - t^2}}$

In Problems 63–68, find $f'(x)$ and find the equation of the line tangent to the graph of f at the indicated value of x .

63. $f(x) = x(4 - x)^3; x = 2$

64. $f(x) = x^2(1 - x)^4; x = 2$

65. $f(x) = \frac{x}{(2x - 5)^3}; x = 3$

66. $f(x) = \frac{x^4}{(3x - 8)^2}; x = 4$

67. $f(x) = \sqrt{\ln x}; x = e$

68. $f(x) = e^{\sqrt{x}}; x = 1$

In Problems 69–74, find $f'(x)$ and find the value(s) of x where the tangent line is horizontal.

69. $f(x) = x^2(x - 5)^3$

70. $f(x) = x^3(x - 7)^4$

71. $f(x) = \frac{x}{(2x + 5)^2}$

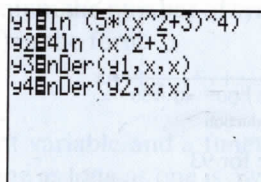
72. $f(x) = \frac{x - 1}{(x - 3)^3}$

73. $f(x) = \sqrt{x^2 - 8x + 20}$

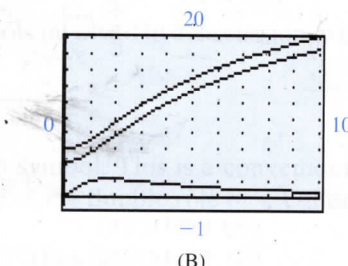
74. $f(x) = \sqrt{x^2 + 4x + 5}$



75. Suppose a student reasons that the functions $f(x) = \ln[5(x^2 + 3)^4]$ and $g(x) = 4 \ln(x^2 + 3)$ must have the same derivative, since he has entered $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ into a graphing calculator, but only three graphs appear (see the figure). Is his reasoning correct? Are $f'(x)$ and $g'(x)$ the same function? Explain.



(A)

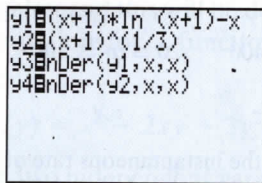


(B)

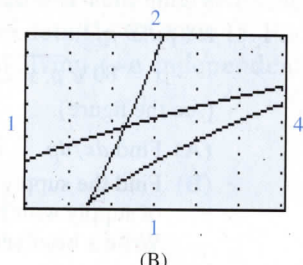
Figure for 75



76. Suppose a student reasons that the functions $f(x) = (x + 1) \ln(x + 1) - x$ and $g(x) = (x + 1)^{1/3}$ must have the same derivative, since she has entered $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ into a graphing calculator, but only three graphs appear (see the figure). Is her reasoning correct? Are $f'(x)$ and $g'(x)$ the same function? Explain.



(A)



(B)

Figure for 76

C In Problems 77–92, find each derivative and simplify.

77. $\frac{d}{dx} [3x(x^2 + 1)^3]$

78. $\frac{d}{dx} [2x^2(x^3 - 3)^4]$

79. $\frac{d}{dx} \frac{(x^3 - 7)^4}{2x^3}$

80. $\frac{d}{dx} \frac{3x^2}{(x^2 + 5)^3}$

81. $\frac{d}{dx} \log_2(3x^2 - 1)$

82. $\frac{d}{dx} \log(x^3 - 1)$

83. $\frac{d}{dx} 10^{t^2+x}$

84. $\frac{d}{dx} 8^{1-2x^2}$

85. $\frac{d}{dx} \log_3(4x^3 + 5x + 7)$

86. $\frac{d}{dx} \log_5(5^{x^2-1})$

87. $\frac{d}{dx} 2^{x^3-x^2+4x+1}$

88. $\frac{d}{dx} 10^{\ln x}$

89. $\frac{d}{dx} \frac{2x}{\sqrt{x-3}}$

90. $\frac{d}{dx} \frac{x^2}{\sqrt{x^2+1}}$

91. $\frac{d}{dx} \sqrt{(2x-1)^3(x^2+3)^4}$

92. $\frac{d}{dx} \sqrt{\frac{4x+1}{2x^2+1}}$